MAY 20 1923

58—12

Proceedings of the American Academy of Arts and Sciences.

Vol. 58. No. 12. — May, 1923.

THE MINIMUM AUDIBLE INTENSITY OF SOUND.

By Clifford M. Swan.

(Continued from page 3 of cover.)

VOLUME 58.

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Received January 2, 1923. Presented by Theodore Lyman.

The absolute energy value at the threshold of audibility has engaged the attention of experimenters for a number of years. It is of interest in the domain of experimental psychology, and of late has figured in researches in telephone engineering. It is also of importance in the work of architectural acoustics, since the laws governing reverberation as laid down by W. C. Sabine [1] make use of the minimum audible sound as the unit of intensity. Previous researches having shown wide discrepancies and uncertainties in results, the investigation here reported was undertaken several years ago to establish more reliable values if possible. The author is indebted to the late Professor Sabine for his assistance in making check observations in all of the determinations.

Previous Work.

In reviewing previous work in this field, we find that the first quantitative determination was made by Toepler & Boltzmann [2] in 1870. The variation in density at the node at the end of a closed organ pipe, blown at constant pressure, was measured by an optical interference method, with a stroboscopic device to slow down the rapidly shifting interference pattern and render it visible to the eye. The amplitude of vibration at any distance outside the pipe was deduced theoretically in terms of the density variation within, and the distance measured to the point at which the sound became inaudible. No attempt was made to reach an exact result, the experiment being supplementary to another piece of work, and made only to determine the order of magnitude of the quantity sought.

In 1877, Rayleigh [3] attempted to solve the problem from the theoretical amount of energy consumed and the distance at which the sound became inaudible when a whistle of high pitch was blown at constant pressure.

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1 Contributions, Jeff. Phys. Lab., Harvard Univ., 1906, Vol. 4, pp. 1–74. Also, Collected Papers on Acoustics, W. C. Sabine, Harvard Univ. Press, 1922.
2 Pogg. Ann., 1870, Vol. 141, p. 349.
3 Proc. Roy. Soc., 1877, Vol. 26, p. 248.

Wead [4] followed with a series of experiments on tuning forks. An observer was stationed at a fixed distance, and the vibrations of the fork were allowed to die away, the amplitude being noted at the instant the observer ceased to hear the sound. The rate of emission of energy at the moment was calculated and thence the minimum audible energy. A somewhat similar method was employed by Rayleigh [5] a few years later.

Investigations in more recent years were made with telephone diaphragms. The experiments of Rayleigh,[6] Franke,[7] Shaw,[8] and Abraham,[9] along this line are fragmentary and do not call for extended consideration. Zwaardemaker and Quix [10] made an extensive study of the problem between pitches 32 and 12228, using tuning forks for the lower register and organ pipes for the upper. The methods employed give only the relative sensitiveness, and are open to serious criticisms which have been ably discussed by Wien.[11]

The results of all of these investigations are inconsistent, a fact which is not surprising when the sources of error are considered. The experiments in the first group were carried on in the daytime, out-of-doors, and all of the observers speak of the annoyance caused by wind, birds and other extraneous sources of sound. As anyone will appreciate who has tried such experiments, the slightest disturbing sound is sufficient to preclude accuracy in measurements, the sound being so faint near the threshold value that it is entirely obliterated unless absolute quiet prevails. Further criticism may be made of the use of the inverse square law in the computations. It is not conceivable that this law should hold, even in an open field, as the absorbing power of the surface of the ground alone is sufficient to render it invalid. Beyond these obvious difficulties, which amply account for inconsistencies in the results, note should be made of the fact that, contrary to assumption, all of the energy consumed by a sounding body does not reappear as sound, thus furnishing another source of error in many of the cases mentioned.

Some interesting work has been done by Wien. The early investi-

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4 Sill. Jour., 3rd Series, 1883, Vol. 26, p. 177; and 1891, Vol. 41, p. 232.
5 Phil. Mag., 1894, Vol. 38, p. 365.
6 Phil. Mag., 1894, Vol. 38, p. 285.
7 Elektrotech. Zeit., 1890, Vol. 11, p. 288; and Jour. Inst. Elec. Engrs., Vol. 19, p. 754.
8 Proc. Roy. Soc., Series A, 1905, Vol. 76, p. 360.
9 Comptes Rendus, 1907, Vol. 144, p. 1099.
10 Arch. f. Anat. und Physiol., Suppl., 1902, p. 367.
11 Pflueger's Arch. f. Gesammte Physiol., 1903, Vol. 97, p. 48.

gation of that experimenter[12] consisted in the measurement of the pressure variations in a sound wave at the limit of audibility by means of a sensitive resonator. This method was finally discarded on account of the large errors of observation inherent in it, and another line of attack adopted.[13] The relative sensitiveness of the ear for tones of different pitch ranging from 50 to 16,000 vibrations was determined by two methods. One of these involved a direct measurement of the relative minimum intensity by means of the current necessary to produce a sound in a telephone diaphragm pressed against the ear and acting like a piston in the outer chamber of the ear. The other method consisted in the calculation of the relative intensity from the current necessary to produce audibility at a distance 30 centimeters from the telephone. In the latter case the diaphragm was placed in an aperture in the extended plane surface, so that it was theoretically possible to calculate the energy at any point in the neighborhood for any amplitude of the diaphragm. By the second method the absolute as well as the relative values of the minimum audible sound for different pitches were determined. The results of the two methods give curves for the *relative* sensitiveness with differing pitch which are in close agreement. The *absolute* values are, however, much smaller than those obtained by any other experimenter. Rayleigh[14] doubted the validity of the Wien sensitiveness curve, owing to certain experiments of his own, but in general Wien's results were, for many years, accepted as correct.

In 1921, Kranz[15] employed a thermophone pressed against the ear as a calculable source of sound, and by gradually reducing the current until he could no longer detect the opening or closing of the circuit, he determined the minimum audible energy. His results have been published for only one pitch.

Lane[16] has conducted an investigation over the upper range of pitch, above 2000, using a Hewlett tone generator as a source. His experiments were carried on at night, out-of-doors, and the disturbance due to the noise of insects is mentioned. His results are therefore open to the same criticism as the work of the early experimenters already noted. He finds the minimum audible intensity to remain practically constant between pitches 2000 to 14000.

More recently, Fletcher and Wegel[17] have completed an extensive

12 Wied. Ann., 1889, Vol. 36, p. 834.
13 Pflueger's Arch. f. Gesammte Physiol., 1903, Vol. 97, p. 1.
14 Phil. Mag., 1907, Vol. 14, p. 596.
15 Physical Review, Second Series, March, 1921, Vol. 17, p. 384.
16 Phys. Rev., Second Series, May, 1922, Vol. 19, p. 492.
17 Phys. Rev., Second Series, June, 1922, Vol. 19, p. 553.

research using a special air damped telephone receiver pressed to the ear, with an alternating current from a vacuum tube oscillator, reduced by an attenuator until the sound was just audible. Their results cover a range of pitch from 60 to 4000 cycles.

Table 1 gives a summary of the results of the principal investigations already mentioned, and shows their manifest discrepancies. The figures give the energy in ergs per square centimeter per second.

Aside from the criticisms which each investigator has made of the researches which have preceded his own, the criticism offered from the standpoint of this paper is on the one hand the impossibility of getting conditions for testing minimum audibility out-of-doors, and on the other, the neglect in indoor experiments of the intensifying effect of reverberation and the complication arising from interference. In this research a new method has been adopted which eliminates the one and takes account of the other.

METHOD.

Some work in architectural acoustics by Sabine [18] suggested a new method of approaching the problem. Sabine shows that the time of decay of sound in a closed room after the source has ceased is a function of the initial intensity in the room and of the minimum audible intensity. Hence if the first two of these quantities can be measured experimentally, the last can be calculated. In order to compute the initial intensity in a room, however, it is necessary not only to know certain constants of the room, but also the rate of emission of energy from the sounding source. That form of sound emission which lends itself most readily to theoretical computation is a circular diaphragm, free at the edges, vibrating in an infinite plane wall. In the present investigation, a near approximation to such a source was employed by a circular diaphragm attached to one prong of a tuning fork vibrating in an aperture in a door flush with one wall of a room. By means of electrical connections to be described presently, the sounding source could be damped almost instantly and the time of decay of the residual sound in the room measured by a chronometer.

THEORY.

The rate of emission of energy from a circular diaphragm, free at the edges, vibrating in an infinite plane wall, may be calculated as follows:

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[18] loc. cit.

TABLE 1.

Pitch	Toepler & Boltzmann 1870	Rayleigh 1877	Wead 1883	Wien 1889	Rayleigh 1894	Wien 1902	Kranz 1921	Lane 1922	Fletcher & Wegel (approx.) 1922
60									5.5×10^{-4}
128			8×10^{-5}						3×10^{-5}
181	8.3×10^{-5}								2.3×10^{-6}
200						5.8×10^{-9}			
220				1×10^{-8}					
256			3×10^{-6}		9.3×10^{-7}				1.2×10^{-6}
384			2.6×10^{-6}		5.5×10^{-7}				
400						11×10^{-11}			
440				7.3×10^{-9}					
512			11×10^{-6}		5.5×10^{-7}				7.8×10^{-8}
600						6.6×10^{-11}			
768			16×10^{-6}						
1024			7.1×10^{-6}						2×10^{-8}
1050						3.5×10^{-12}			
2048							2.2×10^{-8}	7×10^{-8}	1.5×10^{-8}
2730		4×10^{-5}						"	"
3500								"	"

The velocity potential at a distance r from a given point in a medium vibrating with simple harmonic motion may be expressed as

$$P \cos (2\pi nt + \epsilon)$$

where n is the pitch of the sound, and P and ϵ are independent of the time t, but are dependent on r. This may be represented by the real part of the imaginary expression

$$\varphi = Pe^{i(kvt+\epsilon)}$$

where

$$k = \frac{2\pi n}{v} = \frac{2\pi}{\lambda}$$

v being the velocity of sound and λ the wave length.
The equation of motion has the general form

$$\frac{\partial^2 \varphi}{\partial t^2} - v^2 \nabla^2 \varphi = \Phi$$

but

$$\frac{\partial^2 \varphi}{\partial t^2} = -k^2 v^2 P e^{i(kvt+\epsilon)}$$
$$= -k^2 v^2 \varphi$$

hence

$$\nabla^2 \varphi + k^2 \varphi + v^2 \Phi = 0$$

The solution of this equation is [19]

$$\varphi = \frac{1}{4\pi v^2} \int\int\int \frac{e^{-ikr}}{r} \Phi \, d\tau$$

When the force has the value Φ_1 and is distributed only over a surface S in the medium,

$$\varphi = \frac{1}{4\pi v^2} \int\int \frac{e^{-ikr}}{r} \Phi_1 \, dS$$

If the surface is plane, and if $\frac{d\varphi}{dn}$ is the normal velocity of the medium at the surface of the plane, then

$$\varphi = -\frac{1}{2\pi} \int\int \frac{d\varphi}{dn} \frac{e^{-ikr}}{r} dS$$

---

19 Rayleigh, Theory of Sound, Art. 277, (3).

If the force is distributed over a solid plane of infinite extent, double the energy is emitted, and

$$\varphi = -\frac{1}{\pi}\int\int \frac{d\varphi}{dn}\frac{e^{-ikr}}{r}\,dS$$

If we consider the reaction of the air upon the vibrating plane, the rate of emission of energy will be

$$\frac{dW}{dt} = \int\int \frac{d\varphi}{dn}\cdot\delta p\cdot dS$$

where δp is the pressure variation. If σ is the density of the air,

$$\delta p = -\sigma\frac{d\varphi}{dt}$$

$$\frac{dW}{dt} = -\sigma\int\int \frac{d\varphi}{dn}\frac{d\varphi}{dt}\,dS$$

$$= -ik v\sigma\frac{d\varphi}{dn}\int\int \varphi\,dS$$

if $\frac{d\varphi}{dn}$ is constant over the plane.

Then

$$\frac{dW}{dt} = \frac{ikv\sigma}{\pi}\left[\frac{d\varphi}{dn}\right]^2 \Sigma\Sigma \frac{e^{-ikr}}{r}\,dS\cdot dS'$$

This is to be solved for the case where a circular rigid portion of the plane is vibrating with simple harmonic motion, while the remainder of the plane is fixed.

The evaluation of this expression for the case in question has been shown by Rayleigh [20] to be

$$\frac{dW}{dt} = \frac{ikv\sigma}{\pi}\left[\frac{d\varphi}{dn}\right]^2 \left\{\frac{\pi^2}{2k^3}K_1(2ka) - \frac{i\pi^2a^2}{k}\left[1 - \frac{J_1(2ka)}{ka}\right]\right\}$$

where a is the radius of the disc, and J_1 and K_1 are Bessel's functions of the first and second kinds respectively.

If x is the displacement of the disc at time t, and α is its amplitude of vibration,

---

20 Theory of Sound, Vol. II, Art. 302.

$$x = \alpha e^{i(kvt+\eta)}$$

$$\frac{d\varphi}{dn} = \frac{dx}{dt} = ikv\alpha e^{i(kvt+\eta)}$$

$$\frac{d^2x}{dt^2} = -k^2v^2\alpha e^{i(kvt+\eta)}$$

Then

$$\frac{dW}{dt} = \frac{\pi\sigma}{2k^3} \cdot \frac{dx}{dt} \cdot \frac{d^2x}{dt^2} K_1(2ka) + \pi a^2 v\sigma \left[\frac{dx}{dt}\right]^2 \left[1 - \frac{J_1(2ka)}{ka}\right]$$

Taking the real part of $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$, we have

$$\frac{dW}{dt} = \frac{\pi v^3\sigma\alpha}{2} \sin(kvt+\eta) \cos(kvt+\eta) K_1(2ka)$$

$$+ \pi a^2k^2v^3\sigma\alpha^2 \sin(kvt+\eta) \left[1 - \frac{J_1(2ka)}{ka}\right]$$

The first term is entirely periodic and hence disappears after the fork has made a few vibrations and gained its full amplitude. The mean value of $\sin^2(kvt+\eta)$ is $\frac{1}{2}$. Hence, in the steady condition,

$$W = \frac{\pi a^2k^2v^3\sigma\alpha^2 t}{2} \left[1 - \frac{J_1(2ka)}{ka}\right]$$

Hence the average rate of emission of energy is

$$E = \frac{W}{t} = 2\pi^3a^2v\sigma n^2\alpha^2 \left(1 - \frac{vJ_1\left(\frac{4\pi na}{v}\right)}{2\pi na}\right)$$

Sabine [21] has shown that in a closed room of volume V,

$$E = VA\, i'\, e^{At}$$

where i' is the minimum audible energy per cubic unit, and

$$A = \frac{\log_e \frac{I_2}{I_1}}{t_2 - t_1} = \frac{\log_e \left[\frac{\alpha_2}{\alpha_1}\right]^2}{t_2 - t_1}$$

---

21 loc. cit.

Combining these expressions, we have

$$i' = \frac{2\pi^3 a^2 v \sigma n^2 \alpha^2}{V A e^{At}} \left(1 - \frac{v J_1\left(\frac{4\pi n a}{v}\right)}{2\pi n a} \right)$$

In order to find the energy per square centimeter per second, it is necessary only to multiply the value of i' by the velocity of sound v, thus giving the expression

$$E' = i'v = \frac{2\pi^3 a^2 v^2 \sigma n^2 \alpha^2}{V A e^{At}} \left(1 - \frac{v J_1\left(\frac{4\pi n a}{v}\right)}{2\pi n a} \right)$$

from which the results of this investigation were computed. In order to determine A, it is evidently necessary to measure the time of decay of the sound for at least two different amplitudes of the fork.

Apparatus.

The experiments to be described were made in the Constant Temperature Room of the Jefferson Physical Laboratory of Harvard University. This room has a volume of 76 cubic meters and is constructed with heavy double brick walls situated in the sub-basement of the tower of the Laboratory and free from structural connection with the surrounding building. It is situated at some distance from the street and its attendant noises. All observations were made at night under conditions of absolute quiet.

The source of sound consisted of a circular aluminum diaphragm, free at the edges, vibrating in a circular aperture in the door of the room, the diaphragm being attached to one prong of a tuning fork F outside of the door.

Symmetrical conditions in respect to weight of the fork were secured by attaching an exactly similar diaphragm to the other tyne, idle of course as far as producing sound in the room was concerned, since it was located outside of the room. Symmetrical damping conditions were secured by surrounding the second diaphragm with a guard-ring. The air damping of the diaphragms was reduced by tuning to resonance the space between them. This was accomplished by means of a plunger resonator. This device also served the even more important purpose of causing the air in the very small annulus between the

guard-rings and the diaphragms to follow the motion of the latter. The amplitude of vibration of the tyne of the fork was measured by observing through a microscope provided with a micrometer eyepiece the edge of a razor blade attached to the tyne.

The fork *F* was driven electromagnetically by a second fork *D* situated in a distant room to remove it from the range of hearing. On

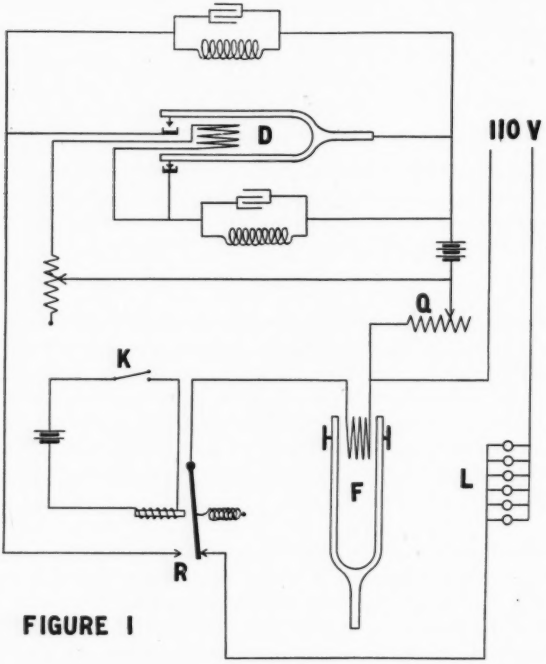

FIGURE 1

account of unequal changes of temperature in the neighborhood of the two forks, it was necessary to tune the driving fork frequently to keep it in unison with the natural period of the sounding fork. The driving fork was operated without intermission, the sounding fork only at the pleasure of the observer in the Constant Temperature Room. The latter fork was controlled by a key *K* in the hands of the observer which closed a circuit through a relay *R*, causing the driving current

to flow through the coils of the fork F and set the latter in vibration. The amplitude of this vibration was governed by a rheostat Q in the driving circuit.

During the course of an experiment, the observer sat within a box placed against the wall of the room, so constructed that only his head was exposed. The aggregate of absorbing material in the room was thus diminished, the residual sound prolonged, and the accuracy of the observations increased. On the other hand, it created a certain difficulty. In an enclosed room the sound establishes an interference system which must be reckoned with in any acoustical investigation. On account of this interference pattern, the observer, just as the sound ceases to be audible, may be either at a point of maximum or of minimum disturbance, or between the two. It is therefore desirable to take a number of observations at different points relative to the interference system in the room, which is obviously impossible if the observer is to be confined within the box. The difficulty was overcome, however, in a manner much more effective than by moving the observer. The interference system itself was made to shift by a pair of large reflectors reaching more than half way across the room, suspended from a horizontal bar, and revolved by clock work about a vertical axis. This served also to eliminate the effect of the reaction of the sound in the room on the source.

The time of decay was measured by a stop-watch so arranged that it began to record at the instant the source of sound was stopped by the key K, and continued to record until stopped by the observer when the residual sound in the room reached the limit of audibility.

Much difficulty was experienced, at first, owing to the continued vibration of the sounding-fork after the current had been broken. An effective method of damping was secured by causing the relay R to close a 110 volt direct current through the coil of the fork F when the driving current was broken. This pulled the prongs of the fork violently together and stopped the vibration instantly. The amount of current was so great, however, as to burn out the coil of the fork in a very short time. The ideal to be sought, evidently, was some means of permitting a momentary flow of a current sufficiently great to damp the fork instantly, followed by an immediate reduction in the current-strength to an amount which could be safely maintained through the coil. Not only this, but the current must be reduced gradually in strength in order to allow the tynes of the fork to resume their normal position slowly and not produce further sound, as would be the case if they were allowed to snap back into place. A way of solving these

difficulties was found by introducing into the 110 volt circuit several 60 watt Mazda lamps, L, connected in multiple. The lamps being cold when the driving current of the fork is broken by the relay, they permit a momentary large damping current to flow, but their rapid increase of resistance as they heat causes exactly the amount and nature of reduction of current-strength which is demanded. The practical operation of this simple but effective device leaves nothing to be desired, and is evidently applicable to many laboratory problems. It was found necessary to use a current from a storage battery as the small periodic fluctuations in the current from a generator were sufficient to produce an audible vibration of the fork. To prevent arcing at the relay on breaking the 110 volt circuit, the gap was immersed in oil.

Observations of the time of decay of the sound were made with four pitches approximately one octave apart, this being the maximum range over which the tuning fork method was found to be practicable. Each series of observations was repeated on several successive nights and checked by two different observers. In all of the experiments an assistant was required to note the amplitude of the sounding fork and to keep it constant. This required incessant watching and adjustment by means of the rheostat on account of small fluctuations in the driving current. With every pitch several different amplitudes of vibration were used for determining the rate of decay in the room, and the mean time of decay corresponding to these amplitudes was obtained from fifty to one hundred chronometer observations in every case. The results are shown in Table 2, where

a = radius of diaphragm
α = amplitude of vibration
n = pitch
t = time of decay of residual sound
A = rate of decay
E' = energy in ergs per sq. cm. per second.

TABLE 2

Observer M				Observer N			
α	t	A	E'	α	t	A	E'
$n = 121$ $a = 2.9$ cm.							
0.040 cm. 0.090 0.182	2.87 3.37 3.86	3.1	5.2×10^{-5}				
0.060 0.180	3.54 4.31	2.9	3.2×10^{-5}	0.060 0.176	3.73 4.59	2.5	8.0×10^{-5}
0.060 0.180	3.31 4.10	2.8	8.1×10^{-5}	0.060 0.180	3.58 4.37	2.8	3.9×10^{-5}
$n = 246$ $a = 2.1$ cm.							
0.004 0.017 0.077	3.09 3.81 4.52	4.2	1.9×10^{-8}	0.003 0.017 0.075	3.10 3.98 4.62	4.3	0.58×10^{-8}
0.005 0.020 0.077	3.10 3.81 4.51	3.9	6.6×10^{-8}	0.005 0.020 0.080	3.14 3.82 4.53	4.0	3.9×10^{-8}
0.005 0.020 0.080	3.14 3.70 4.36	4.6	0.61×10^{-8}				
$n = 493$ $a = 1.1$ cm.							
0.005 0.015 0.045	2.94 3.35 3.86	4.8	4.8×10^{-9}	0.0025 0.015 0.045	2.72 3.40 3.83	5.2	1.0×10^{-9}
0.005 0.015 0.045	2.92 3.36 3.81	4.9	3.4×10^{-9}	0.005 0.015 0.045	2.98 3.39 3.94	4.6	6.9×10^{-9}
0.005 0.015 0.045	3.12 3.58 4.09	4.5	4.8×10^{-9}	0.005 0.015 0.045	3.20 3.68 4.09	5.0	0.7×10^{-9}
$n = 1021$ $a = 0.93$ cm.							
0.0048 0.012	2.97 3.30	5.6	11×10^{-10}	0.0048 0.0144	2.82 3.27	5.8	8.5×10^{-10}
0.0048 0.012	2.98 3.27	6.3	0.97×10^{-10}	0.0048 0.012	2.82 3.14	5.7	15×10^{-10}

Results.

The results are summarized in Table 3. Following Wien, the logarithm of the reciprocal of the minimum energy is used as an index of the sensitivity. In Figure 2, these sensitivity values are plotted against the pitch. Curve 1 shows the result of the present work. Curve 2 shows Wien's final work, and Curve 3 shows that of Fletcher and Wegel. The scattering results of other investigators are also indicated.

TABLE 3.

Pitch	E' Observer M	E' Observer N	E' Weighted mean	$\frac{1}{E'} = S$	$\log_{10}S$
121	5.5×10^{-5}	5.9×10^{-5}	5.7×10^{-5}	1.8×10^{4}	4.3
246	3.0×10^{-8}	2.2×10^{-8}	2.7×10^{-8}	3.7×10^{7}	7.6
493	4.3×10^{-9}	2.9×10^{-9}	3.6×10^{-9}	2.8×10^{8}	8.4
1021	6.0×10^{-10}	12×10^{-10}	9.0×10^{-10}	1.1×10^{9}	9.0

It will be noted that the energy values of the present investigation are somewhat smaller than those obtained by Rayleigh and lie very near the earlier results of Wien, but are considerably larger than Wien's later determinations. This is not surprising as we have already seen that the work of early experimenters was carried on under conditions lacking in quiet which would give too large results, while the later work of Wien is open to criticisms quite as serious operating in the opposite direction.

It is a surprising fact that, in all previous investigations either by physicists or psychologists, the assumption has been made that the sound energy is propagated in spherical or hemispherical waves, obeying the law of inverse squares. Such an assumption is unjustifiable even in experiments conducted out-of-doors, owing to the porosity of the surface of the ground which absorbs the energy and alters its distribution. In a closed room, the sources of error are multiplied on account of reflection from the walls, and here lies the principal criticism to be applied to the later work of Wien.

No matter how small the amount of energy emitted by a source in a closed room, the wave will be reflected from surface to surface many

times before it is dissipated by absorption and transmission. There are three effects of this multiple reflection; first, it increases to a large degree the average intensity of sound within the room, second, it sets

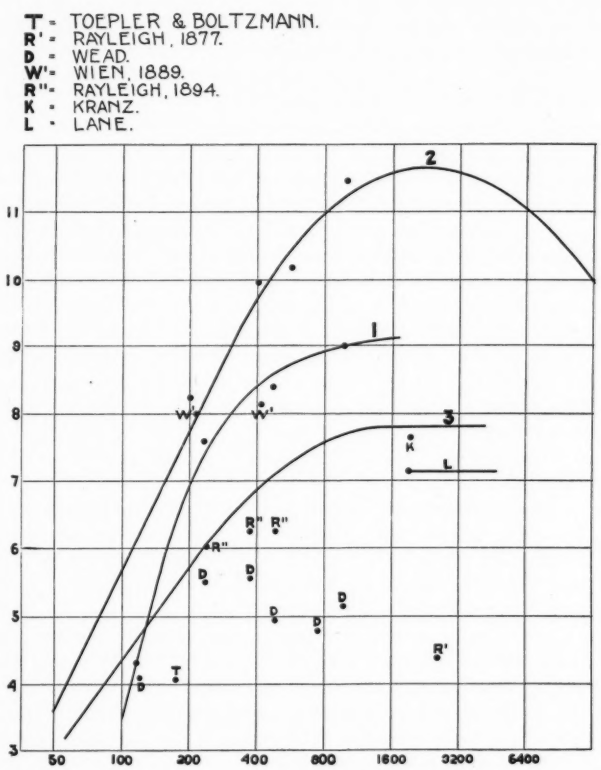

FIGURE 2

up a stationary wave system with definite regions of maximum and minimum intensity, and third, it reacts on the source to change the amount of energy emitted.

In a room such as the lecture hall used by Wien for his experiments, the actual average intensity of sound throughout the room under

normal conditions may be roughly estimated to be at least twenty-five times the average intensity which would exist with totally absorbent walls. Hence, as the current in Wien's telephone was reduced until the sound was just audible at a distance of thirty centimeters, only a fraction reaching the ear came in a direct path from the telephone, the balance being contributed by the reverberation in the room. In other words, at the limit of audibility the current in the telephone was less than would have been required for minimum audibility in a non-reverberant space, thus giving an apparent sensitiveness greater than actually was the case.

Furthermore, with a fixed relation between the source and the ear, as in Wien's experiments, the interference system in the room is not to be disregarded, as the ear cannot conceivably be at a point of average disturbance for all pitches with a constant distance between ear and telephone, and is not likely to be at such a point for even a single pitch.

Add to this the reaction of the room upon the source, making it impossible to determine the amount of energy emitted by the telephone diaphragm or even to compare sources of different pitch, and the wonder is, not that Wien's results are too small, but that there is any possible coincidence that should have caused them to fall on anything approaching a regular curve.

The claim has been made that the wide discrepancies in previous investigations are due to inadequate apparatus and to differences in hearing in the normal ear. These criticisms are not altogether justified, however, and the true causes of the variations are undoubtedly due to the reasons already discussed. Differences between ears, even so-called normal ears, do exist in some cases, no doubt. Even a trained ear may vary from night to night owing to nervous fatigue or a slight cold. But experience for many years with the average results of a number of *trained* observers in measuring reverberation has shown a surprising conformity in threshold sensation. One cannot help but wonder if the differences noted by some experimenters is not due largely to *unskilled* observation. The sense of hearing must be trained just as the sense of taste in the case of a tea taster. It is assuredly impossible to call upon inexperienced students or members of the staff to make a few casual observations with any hope of consistency.

The values of the minimum audible intensity obtained by Fletcher and Wegel are exceedingly interesting, as they are free from most of the sources of error we have noted. They are obtained, however, from the averages of forty-one persons, both men and women, many, if not most of whom, could hardly have been experienced in detecting the

very faint sound near the threshold of audibility. The inevitable result with persons untrained to use their ears is to indicate an end-point long before it is properly reached. This would give the final averages too great a threshold value or too small a sensitivity. The trend of their curve bears this out, as it lies quite close to the values of Rayleigh and of Lane, whose results, as we have seen, are undoubtedly too large owing to other causes.

To sum up, all other investigations in this subject may be grouped in three classes, those carried on out-of-doors, those confined in a closed room with neglect of boundary conditions, and those resulting from too great an admixture of unskilled observation. Each of these has inherent sources of error, tending in the first and third instances to produce too large results, and in the second, too small. The errors in the first case are difficult to eliminate; not so in the other two. It has been the object of this research to undertake such a determination in a closed room with two observers of long experience, using an entirely new method which takes account of the interference and reverberation. The results seem to justify the expectations.

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September, 1922.